Ch01

If you are enrolled
or on the wait list–sign
the roll sheet!
If you are trying to add the
class, add your name!

Measurement

The line between what we know
and what we don’t know.

version 1.5

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Science helps us to explore and expand the edges of our knowledge.
At the edge of our knowledge we know some things incompletely.

Imagine walking into a dark room with a table.

It's not always enough to know that a table exists in the room.
Before we put a box on the table
we need to know the limits of our knowledge.

Where we can say for certain the table exists,
where we can say for certain it doesn't,
and where are uncertain.

Measurements are how we clearly express
the limits of our knowledge.

Ch01

Measurements

- Measurements
  - Are Observations
  - Some are Exact some are Not
    - Precision
    - Accuracy
- Instruments
  - Certainty
  - Finding Limits
    - Digital & Analog
- Significant Numbers
  - In what we observe
  - In what we read
    - The rules for zeroes
  - Scientific Notation
    - The best way to express significant numbers
- Significance in Calculations
  - Calculator Use
    - Scientific Notation
    - Calculators don’t understand significance
  - Round-off to show only significant figures
  - Where to round off after
    - ...Multiplication & Division
    - ...Addition & Subtraction
  - Order of operations

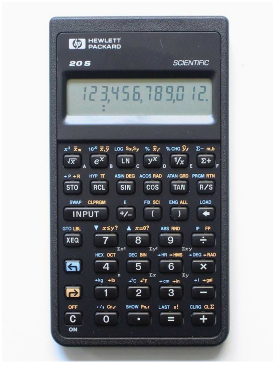
21.5 °C

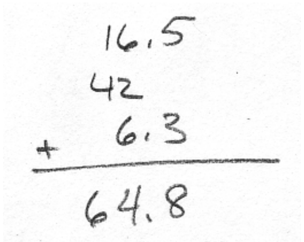
0.873 grams

156.2 mL

23,000,000 feet

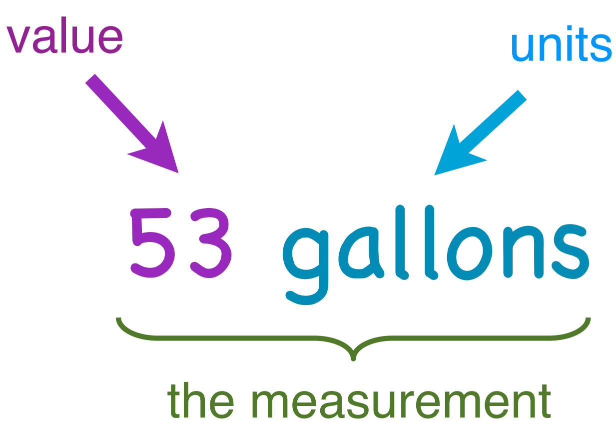
2.30×10^{7} feet





Measurements

- A measurement is a quantitative observation.
    - Observations are facts that can be confirmed by others.
    - Quantitative means it can be expressed in numbers.
- Measurements have two parts:
    - Value (factor)
    - Units (dimension)



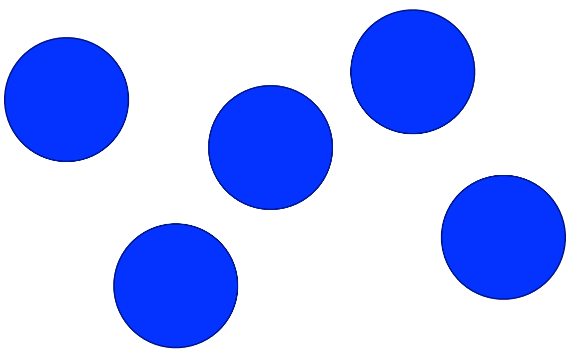
65.7 mph

21.5 °C

1,213 feet

Some Measurements are Exact

- Exact numbers have no uncertainty.
- Exactly 4 means 4, not 4.1 not 3.99999 – exactly 4.
- There are two ways we encounter exact numbers:
  - Exact numbers occur in Counting Operations.



  - Also, Defined Numbers are exact.

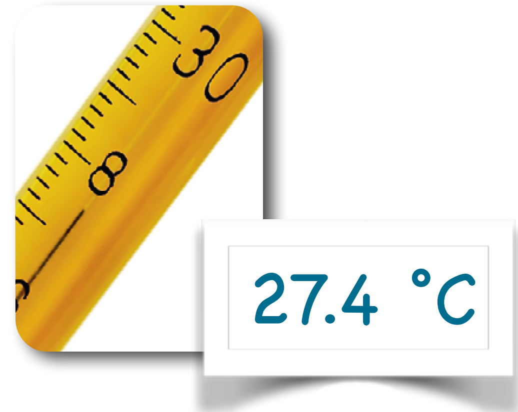
12 inches = 1 foot

100 centimeters = 1 meter

$$12\,\frac{\text{in}}{\text{ft}} \qquad 100\,\frac{\text{cm}}{\text{m}}$$

Most Measurements Are Not Exact

- There’s a limit to what we can say for certain.
- We may know for certain a volume is between 70 mL and 80 mL.
- We may know for certain it’s between 73 mL and 74 mL.
- We can tell it’s more than half way between.
- But what is the “true” value?
- Is it 73.8 mL?
- Is it 73.7 mL?
- Both measurements are valid.
- We just don’t know.
- There’s a limit to what we know.
- It’s important we don’t claim more data than what we know.
- ... and it’s important we don’t fail to claim data we do know.
- Measurements that are not exact can vary.
- They can vary by precision and by accuracy.



What is precision?

1) The text book is 10 inches tall.

2) The text book is 10.7 inches tall.

3) The text book is 10.74 inches tall.

4) The text book is 10.7456735 inches tall.

Increasing Precision

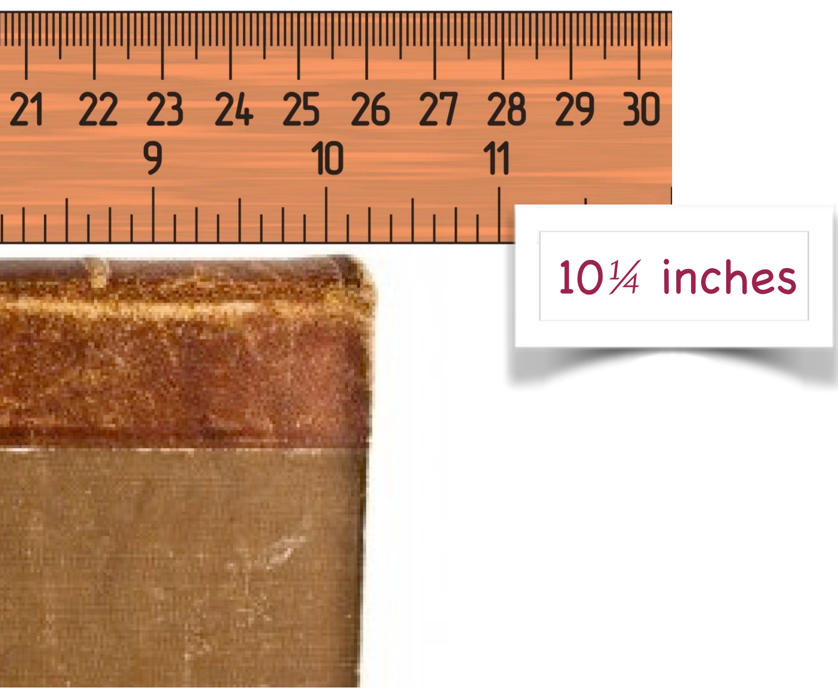
Precision is how many digits (figures) are in the measurement.

(using a ruler the book measures to be 10 1/4 inches tall)

Let’s talk about accuracy…

What is accuracy?

- Our goal is to find measurements we can trust.
- We trust a number if someone else can verify it.
- Accurate measurements are measurements that can be reproduced (and therefore verified).
- A measurement is accurate if it is reproducible.



Accurate?

✔ 1) The book is 10 inches tall.

✗ 2) The book is 10.7 inches tall.

✔ 3) The book is 10.2 inches tall.

✔ 4) The book is 10.24 inches tall

✗ 5) The book is 10.2434625795283 inches tall.

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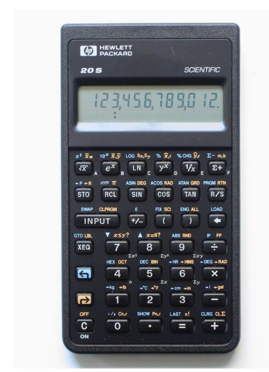
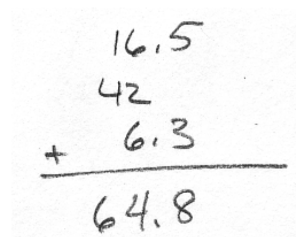
156.2 mL

23,000,000 feet

2.30×10^7 feet

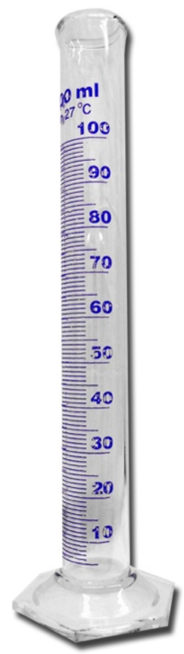


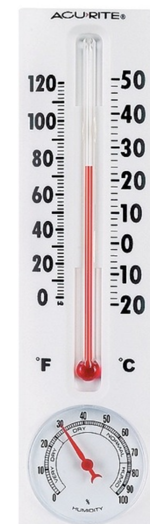


Reading Instruments

- We use instruments to take measurements.
- Instruments have limits.
- It's your job to know your instrument.
- It's your job to know how precise your instrument can be, while still being accurate.
- Don't record measurements with more precision than you trust!
- We'll give you some rules to help.

Analog





Digital

Recording Measurements

All measurements have three parts:

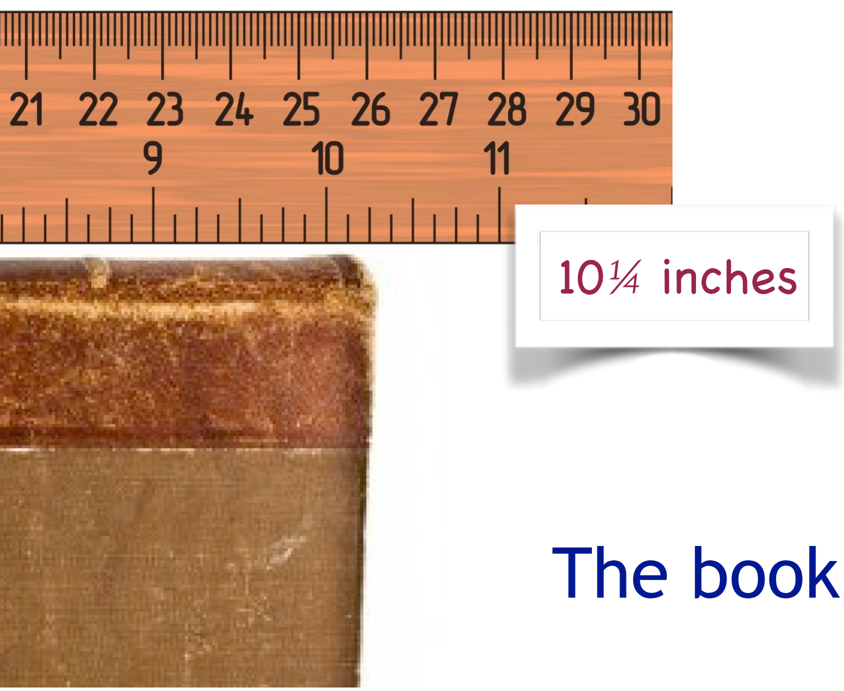
The part we're certain is right.

The part we're uncertain is right.

One figure that's uncertain but can be estimated.

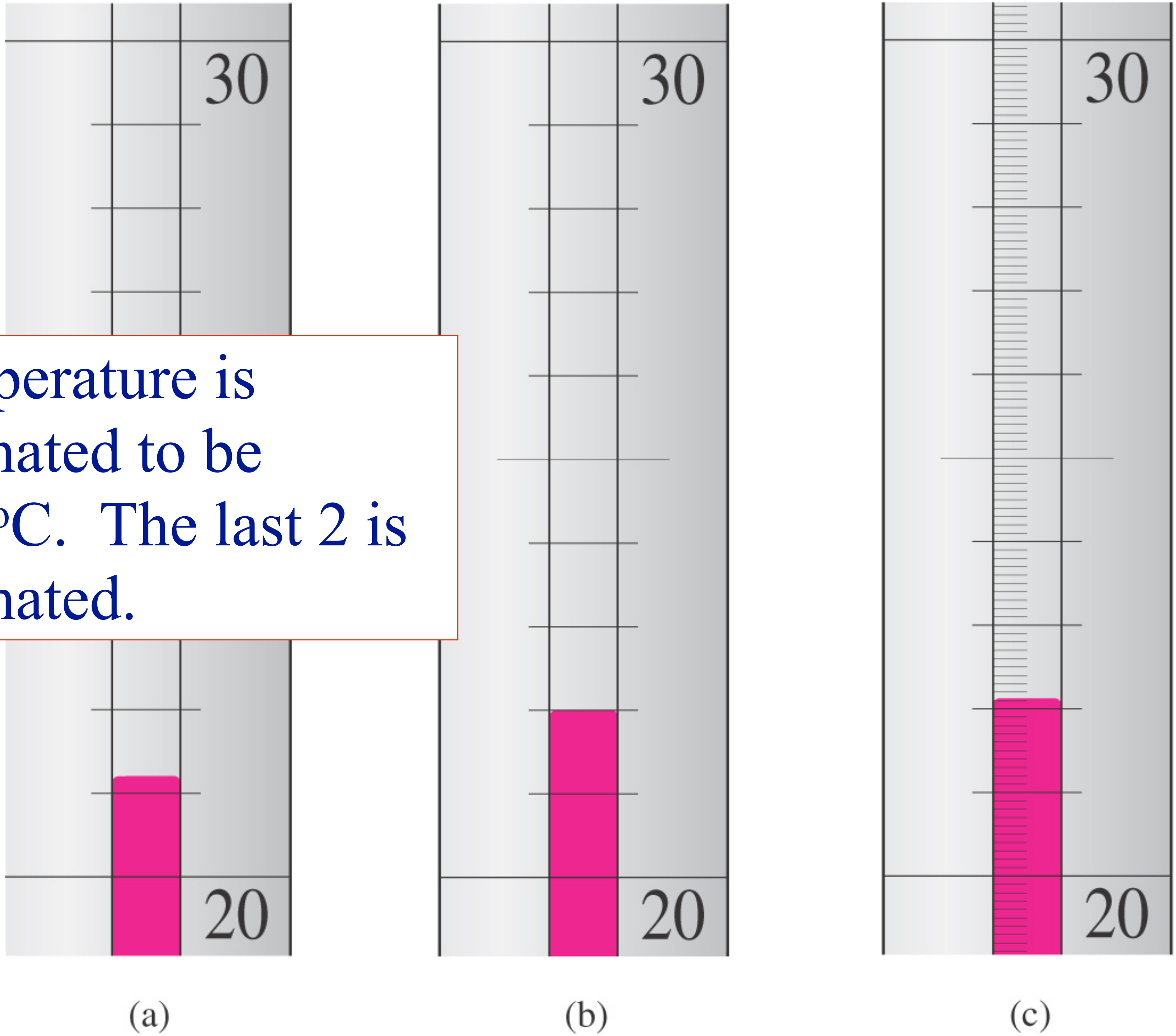
- For a digital instrument, look for a fluctuating digit.
  (the one that flickers back and fourth between 5, 6, 7 …)
- That's where you need to estimate!
- Everything before that is certain.
- Everything after that is uncertain.
- Record only your estimate and what is certain.

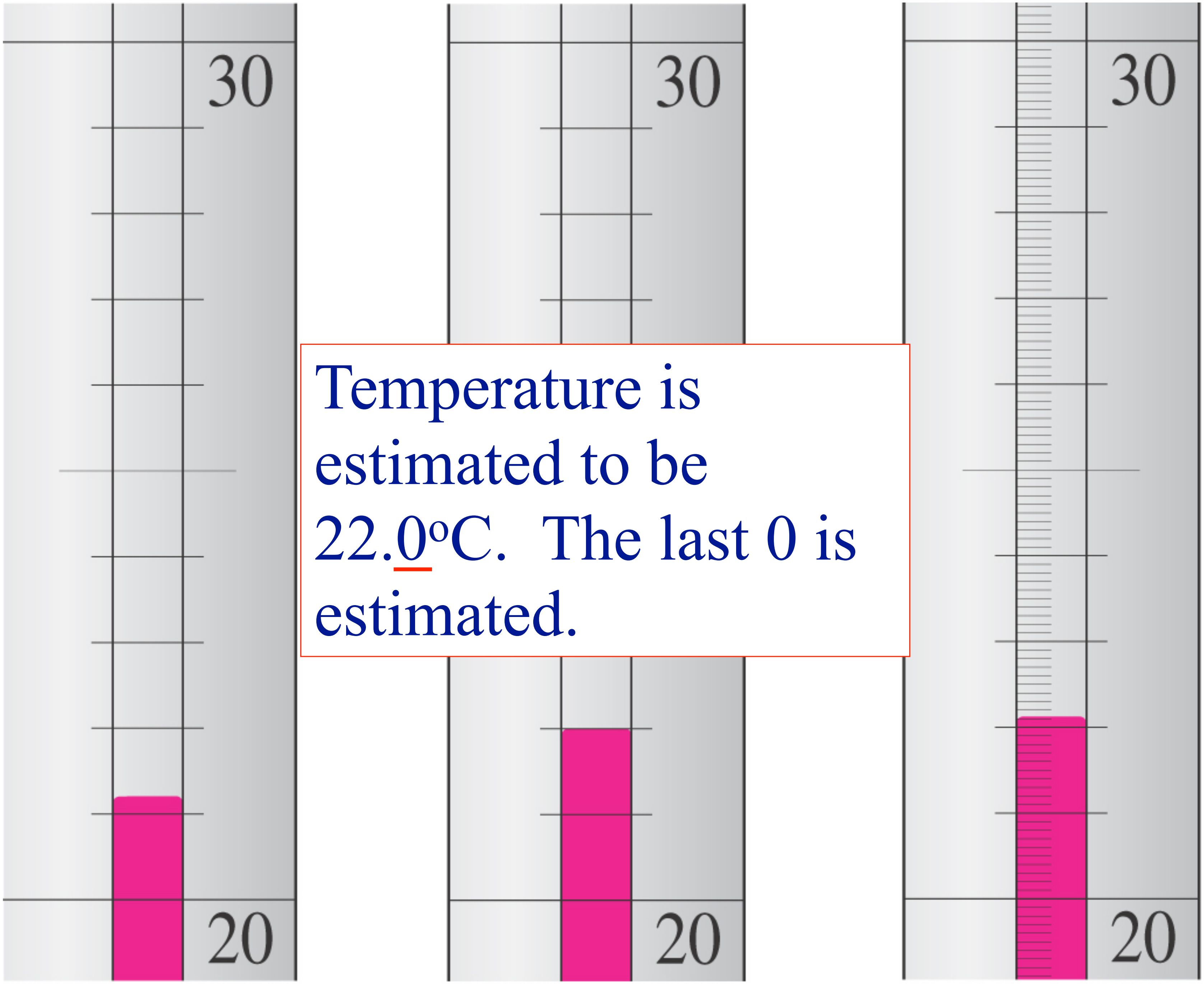
- For an analog instrument, look at the divisions.
  (analog instruments are things like rulers, thermometers, flasks, etc)
- Each one has a smallest division.
- The place between the smallest divisions is where you need to estimate!
- Everything after that is uncertain.
- Record only your estimate and what is certain.



The book is 10.2434625795283 inches tall.

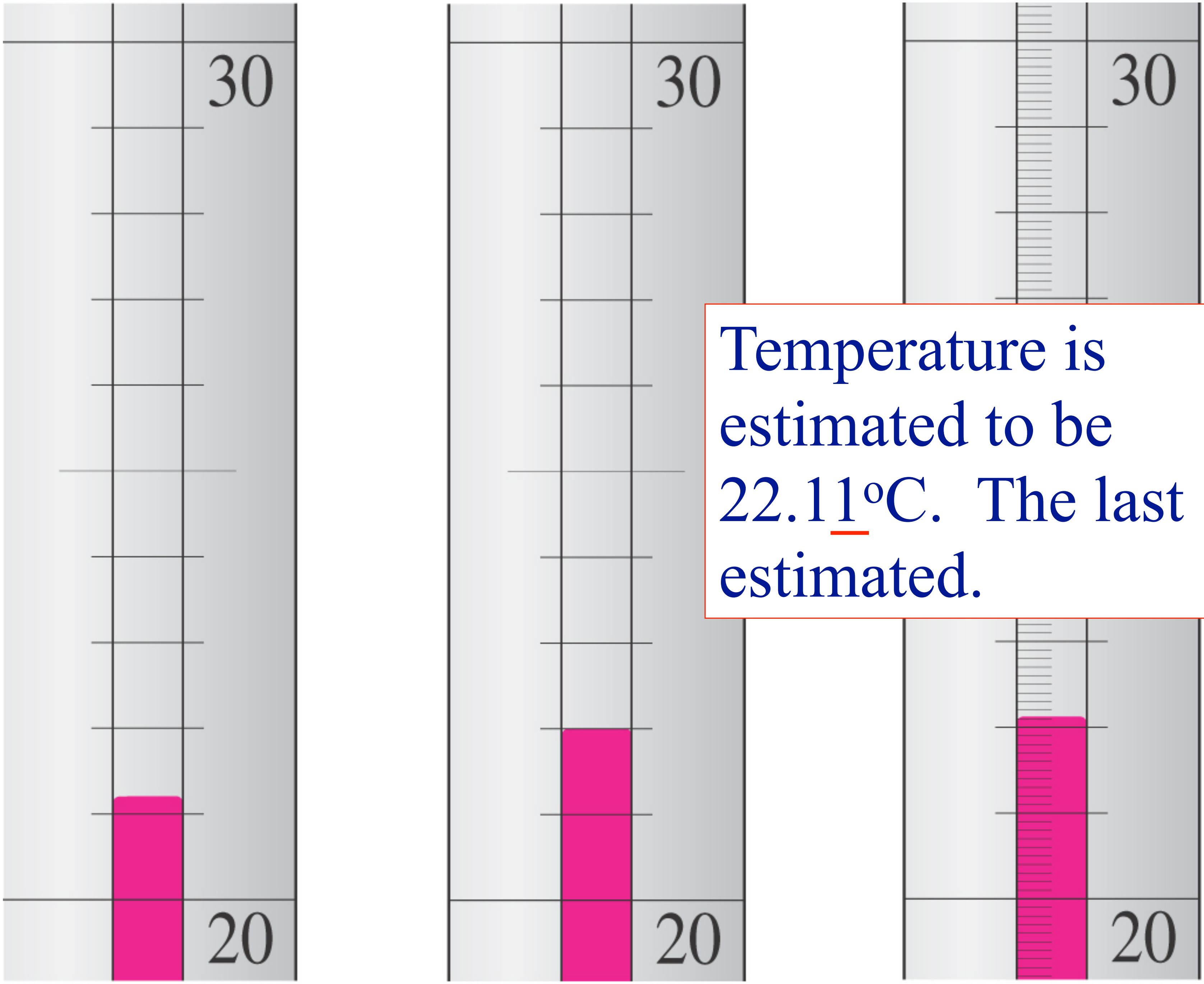
Temperature is estimated to be $21.\underline{2}\,^{o}C$. The last 2 is estimated.





(a) (b) (c)

Temperature is estimated to be 22.0°C. The last 0 is estimated.



(a) (b) (c)

Temperature is estimated to be 22.11°C. The last 1 is estimated.

1.5782458 GRAMS

1.5783279 GRAMS

1.5782132 GRAMS

1.5783456 GRAMS

1.5781999 GRAMS

1.5783756 GRAMS

certain | uncertain

can estimate

Weight of my sample is: 1.5782 grams

The other digits are noise
-- they have no significance!

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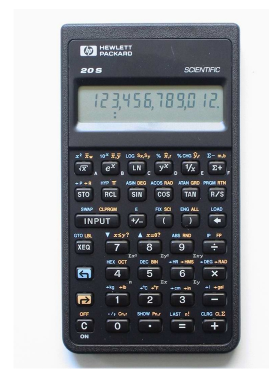
21.5 °C

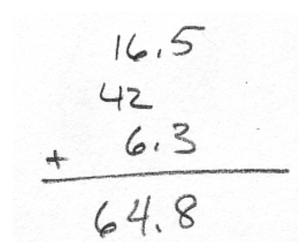
0.873 grams

156.2 mL

23,000,000 feet

2.30×10^7 feet





Recording Measurements

Record:

10.24 inches

- The number of digits that are known plus one estimated digit are considered significant in a measured quantity
- If we take the measurement ourselves it's clear – we know what part is certain.
- If we read someone else's recorded measurement, we trust they only recorded the significant figures.
- Unless there are zeroes.
- There's a problem with zeroes.
- Zeroes might be significant figures.
- But they could also just be place holders.
- If all you have is the number,
  - – you don't know whether to trust the zeroes.

significant | not significant

The book is 10.24|34625795283 inches tall.

A Problem with Zeroes

23,101,179 FEET

23,059,320 FEET

23,114,371 FEET

23,042,998 FEET

23,032,613 FEET

certain uncertain

23,000,013 FEET

23,000,011 FEET

23,000,002 FEET

23,000,009 FEET

23,000.001 FEET

certain uncertain

Length of sample A is: 23,000,000 feet

Length of sample B is: 23,000,000 feet

If you just see the final numbers, you can't tell how many zeroes are significant -- you can't tell how many zeroes to trust!

...oes

Length of sample A is: 23,000,000 feet

Length of sample C is: 23,000,000 feet

Length of sample B is: 23,000,000 feet

We need some rules to tell us how many digits to trust.

Knowing when to trust a digit.

1) All nonzero digits are significant.
2) A zero is significant when it is between nonzero digits.
3) A zero is not significant when it is before the first nonzero digit.
4) A zero is not significant when it is at the end of a number *without a decimal point.*

Knowing when to trust a digit.

1) All nonzero digits are significant.

6.17 °C 46.2 miles per hour

12,213 feet 175 gallons

There is no reason to write down any of those digits if the guy writing them didn't want to claim he was certain or at least making an estimate of that value.

So we assume any non zero digit is significant.

Knowing when to trust a digit.

2) A zero is significant when it is between nonzero digits.

1.07 °C 50.2 miles per hour

21,003 feet 105 gallons

None of these zeroes are needed to show where the decimal point is. The only reason to write these zeroes is to show a digit greater than 9 and less than 1.

So we assume a zero is significant when it's between two nonzero digits.

Knowing when to trust a digit.

3) A zero is not significant when it is before the first nonzero digit.

0.07 °C .052 miles per hour

Zeroes before the first nonzero digit just exist to show us where the decimal point is. They are not significant to the measurement, they're just placeholders.

So we assume a zero is not significant when it's before the first nonzero digit.

Knowing when to trust a digit.

4) A zero is not significant when it is at the end of a number ***without a decimal point.***

21,000 feet 100 gallons

Zeroes at the end of the number *could go either way*. They could have been measured or they could just be placeholders. We don't know, we can't trust them.

So we assume a zero at the end of a number is not significant, **UNLESS...**

Knowing when to trust a digit.

4) A zero is not significant when it is at the end of a number ***without a decimal point.***

1.00 °C　　52.0 miles per hour

100. gallons

If the zeroes are not needed as placeholders or the decimal point wasn't needed, the guy must have written it for a reason — the zeroes must be significant.

So we assume a zero at the end of a number is significant, **if there is a decimal.**

Knowing when to trust a digit.

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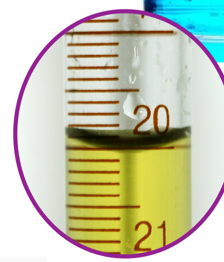
21.5 °C

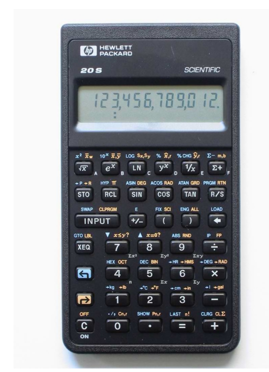
0.873 grams

156.2 mL



23,000,000 feet

2.30×10^7 feet



16.5
42
+ 6.3
64.8

Scientific Notation

Very large and very small numbers are often encountered in science.

Very large and very small numbers like these are awkward and difficult to work with.

602200000000000000000000

0.00000000000000000000625

Scientific Notation

- A method for representing these numbers in a simpler form is called scientific notation.
- Move the decimal point in the original number so that it is located after the first nonzero digit.
- Only keep significant digits.
- Follow the new number by a multiplication sign and 10 with an exponent (power).
- The exponent is equal to the number of places that the decimal point was shifted.

602200000000000000000000

6.022×10^{23}

0.00000000000000000000625

6.25×10^{-21}

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0.053 mL → 5.3×10^{-2} mL

320 grams → 3.2×10^{2} grams

Scientific Notation

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- Only keep significant digits.
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0.017 °C → 1.7×10^{-2} °C

12,213 feet → 1.2213×10^{4} feet

2100 gallons → 2.1×10^{3} gallons

210.0 mph → 2.100×10^{2} mph

The value of Scientific Notation is Clarity.

It allows you to report the significant digits in your measurement without using placeholder zeroes that might confuse folks.

23,101,179 FEET

23,059,320 FEET

23,114,371 FEET

23,042,998 FEET

23,032,613 FEET

certain uncertain

23,000,013 FEET

23,000,011 FEET

23,000,002 FEET

23,000,009 FEET

23,000.001 FEET

certain uncertain

Length of sample A is: ~~23,000,000 feet~~

2.30×10^7 feet

Length of sample B is: ~~23,000,000 feet~~

2.300000×10^7 feet

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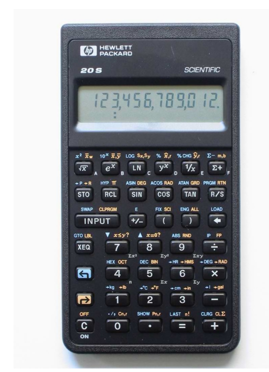
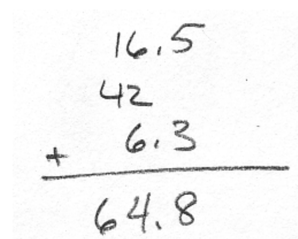
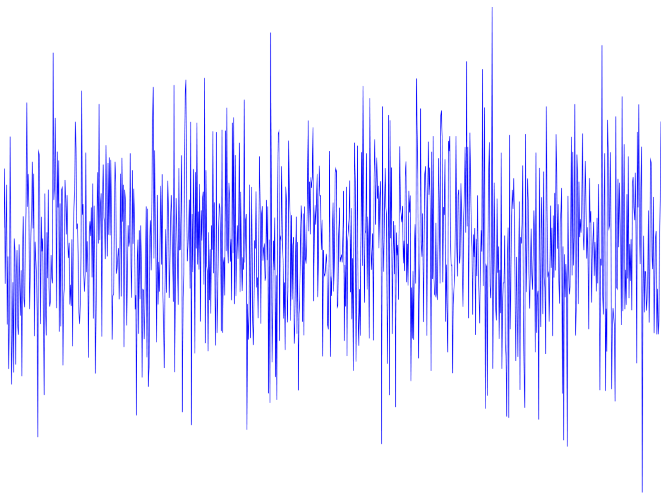




Significant Figures in Calculations

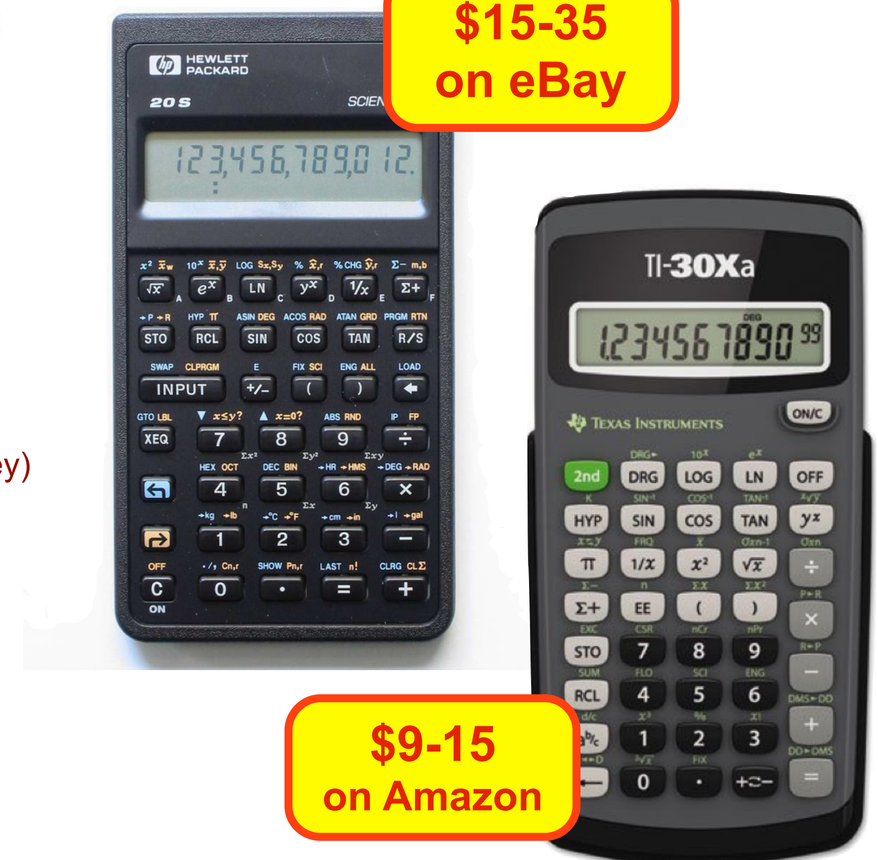
- Instruments produce noise in reporting measurements
- It's your job to distinguish between the noise and the significant digits
- Calculators are a special kind of instrument
  - Calculators also produce noise and you need to find the significant digits in that output
  - Not all calculators are the same.
  - You need the right tool for the right job
- The results of a calculation based on measurements cannot be more precise than the least precise measurement.
  - ... And there are tricks to operating it properly.
    - How to enter scientific notation
    - How to read the display
    - What order to do multi step operations
    - And different ways to read the results
      - Don't read the results of a multiplication the same as an addition



Calculator Check

Must do scientific notation.
(must have an EE or E or Exp key)

Cell phones/PDAs are not acceptable.



$15-35 on eBay

$9-15 on Amazon

Best choice:
a simple calculator with
log and scientific notation keys
- HP 20s (27s or 42s also good)
- Texas Inst TI-30Xa (least expensive)

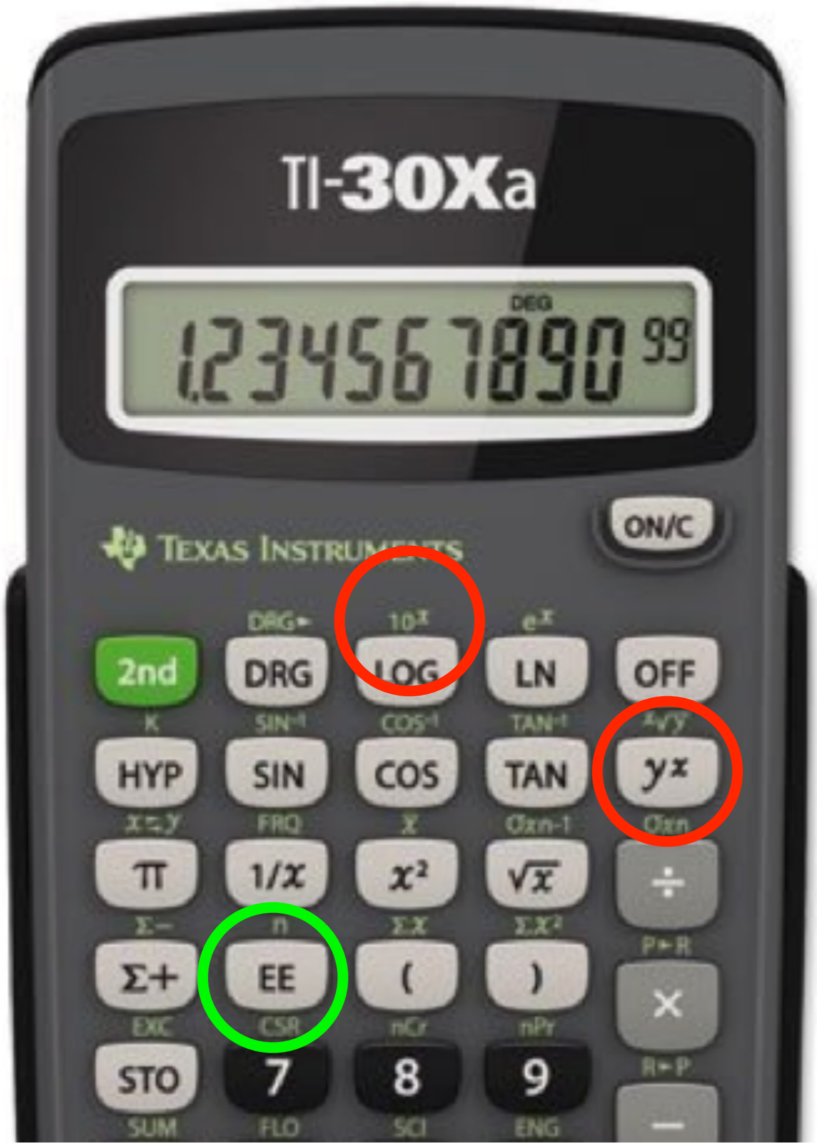
Graphing calculators are bad — they are expensive, hard to use and may trip you up on an exam.

Don't buy one. If you already have one and *__know how to use it well__*, it's acceptable.

CAUTION:
Chem lab calculators are like boxers, they don't stay pretty for long.

__Do not spend big money on any calculator, it might take an acid bath tomorrow!__

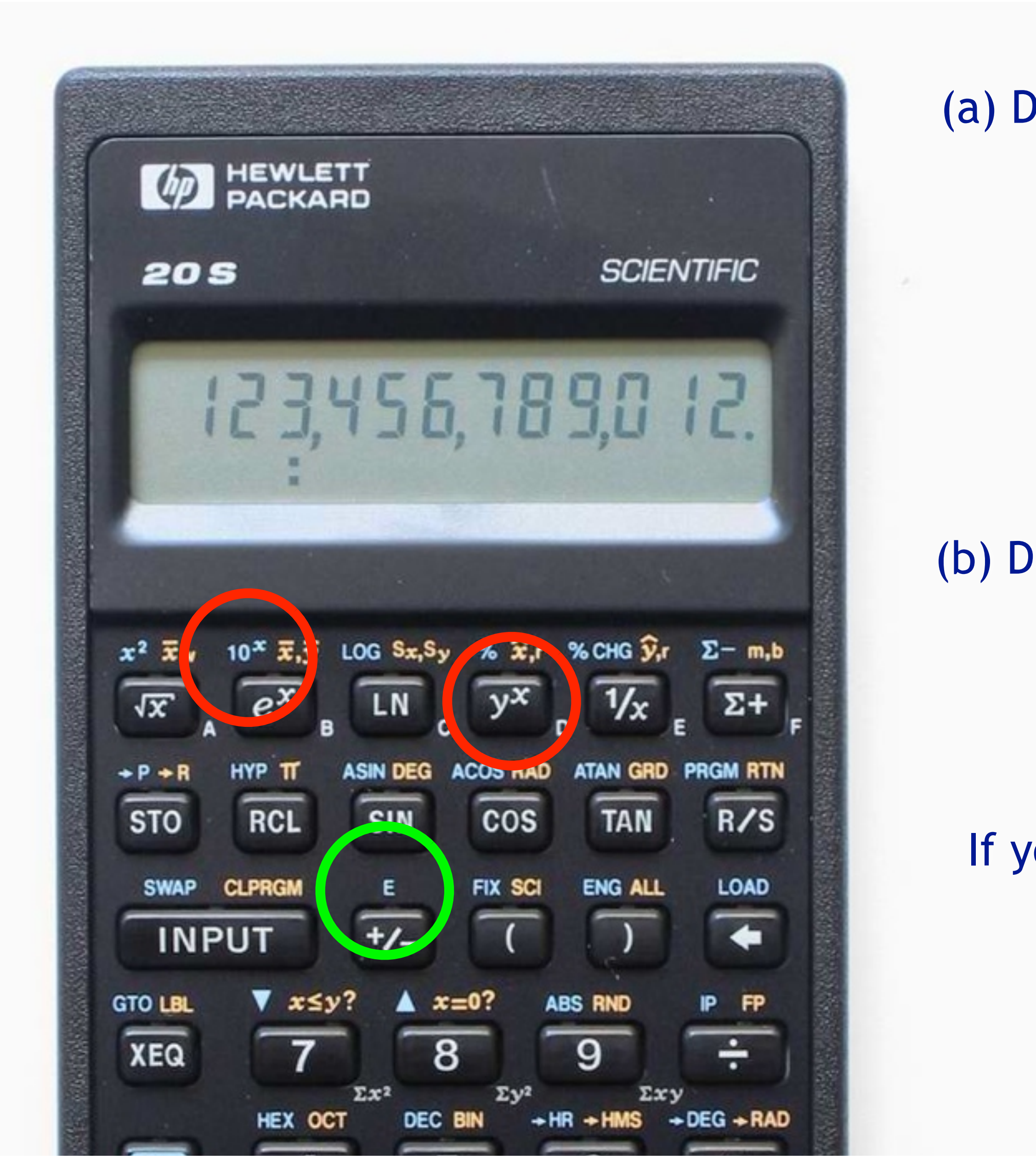
Entering Scientific Notation



- To enter scientific notation look for a key that says:
  - E, EE, Exp, or x10^X
  - Do not use the keys labeled 10^X or y^X
- Type "2.5 E 4" for 2.5 x10^4
- Type "2.5 E 4 =" check that it responds:

  25000

Entering Scientific Notation



(a) Do this calculation: $\dfrac{20.8}{5 \times 10^{3}} =$

The answer is 0.00416 or 4.16×10^{-3}

If you got 4,160 you are using the *wrong* key.

(b) Do this calculation: $\dfrac{1}{3} =$

The answer is 3.33333333E-1

or $3.33333333 \times 10^{-1}$

If you got "1/3" or less than a full screen of "3's" your calculator is:

(1) not ideal for this class and/or

(2) in the wrong display mode

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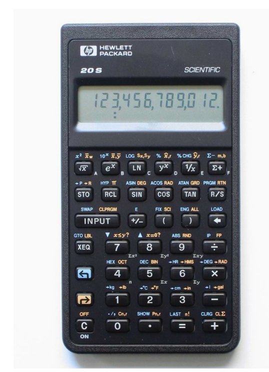

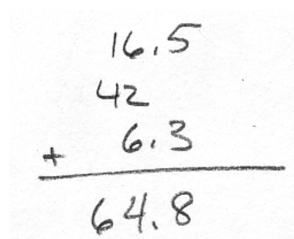

Rounding off the Noise

- Your calculator doesn't know if the number you entered is exact or a measurement with finite significant figures — there's no way of telling the calculator.
- The calculator assumes everything is exact, it assumes the 10 you typed is exactly 10 with infinite significant figures. Not 10 or 10.0 or 10.0000.
- So the calculator often reports extra digits that we know cannot be trusted.
- It is necessary to drop these extra digits so as to express the answer to the correct number of significant figures.
- When digits are dropped, the value of the last digit retained is estimated by a process known as rounding off numbers.

Rounding Off the Estimated Digit

- Rule 1. When the first digit after those you want to retain is 0,1,2,3 or 4 – that digit and all others to its right are dropped. The last digit retained is not changed.
- Rule 2. When the first digit after those you want to retain is 5, 6, 7, 8 or 9 – that digit and all others to its right are dropped. The last digit retained is increased by 1.

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round to 3 significant figures

0.017534 → 0.0175

12,213 → 12200 or 1.22×10^4

12,257 → 12300 or 1.23×10^4

92.168246 → 92.2 or 9.22×10^1

Rounding Off the Estimated Digit

- Rule 1. When the first digit after those you want to retain is 0,1,2,3 or 4 — that digit and all others to its right are dropped. The last digit retained is not changed.
- Rule 2. When the first digit after those you want to retain is 5, 6, 7, 8 or 9 — that digit and all others to its right are dropped. The last digit retained is increased by 1.

round to 3 significant figures

100.235 → ~~100~~ 100. ✔

82,035 → ~~82,000~~ or ~~82,000.~~

8.20×10^4 ✔

Sometimes the **only** way to show the correct sig figs is with scientific notation.

Measurements

- Measurements
  - Are Observations
  - Some are Exact some are Not
    - Precision
    - Accuracy
- Instruments
  - Certainty
  - Finding Limits
    - Digital & Analog
- Significant Numbers
  - In what we observe
  - In what we read
    - The rules for zeroes
  - Scientific Notation
    - The best way to express significant numbers
- Significance in Calculations
  - Calculator Use
    - Scientific Notation
    - Calculators don't understand significance
  - Round-off to show only significant figures
  - How to round off after
    - ...Multiplication & Division
    - ...Addition & Subtraction
  - Order of operations

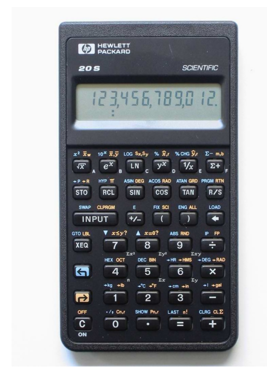
21.5 °C

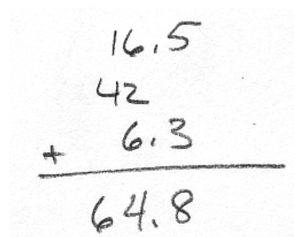
0.873 grams

156.2 mL

23,000,000 feet

2.30 x 10^7 feet





So where do we round off?

to keep our sig figs accurate

- Multiplication & Division
  - The answer must contain the same number of significant figures as in the measurement that has the least number of significant figures.

$17 \times 42 \times 6.349 = 4{,}533.186$

$17 \times 42 \times 6.25 = 4{,}462.5$

2 s.f. 2 s.f. 2 s.f.

$17 \times 42 \times 6.3 = 4{,}498.2$

$= 4.5 \times 10^3$

2 s.f.

- Addition & Subtraction
  - The results of an addition or a subtraction must be expressed to the same precision as the least precise measurement.

same thing said another way:

  - The result must be rounded to the same number of decimal places as the value with the fewest decimal places.

$$\begin{array}{r} 17 \\ 42 \\ +\ 6.3 \\ \hline 65.3 \end{array}$$

$= 65$

$$\begin{array}{r} 16.5 \\ 42 \\ +\ 6.3 \\ \hline 64.8 \end{array}$$

+ & − has different rules than × & ÷

So where do we round off?

to keep our sig figs accurate

+ & −
has different rules than
× & ÷

- Compound Operations
  - If the equation has both multiplication/division and also has addition/subtraction carefully follow the order of operations from basic Algebra:
    - Rule 1: First perform any calculations inside parentheses.
      - Anything above or below a fraction bar is always in parenthesis.
      - Anything raised to a power is always in parenthesis.
    - Rule 2: Next perform all multiplications and divisions, working from left to right.
    - Rule 3: Lastly, perform all additions and subtractions, working from left to right.

$$\frac{a+b}{c}\times\left(a^3-d\right)$$

$$\frac{\left(a+b\right)}{c}\times\left(\left(a^3\right)-d\right)$$

Ex 1:

$$\left(53.6+79.4\right)\times 1.503 =$$

$$\left(53.6+79.4\right)\times 1.503$$

$$= 133.0 \times 1.503$$

$$= 199.899$$

$$= 199.9$$

$$\begin{array}{r} 53.6 \\ +\ 79.4 \\ \hline 133.0 \end{array}$$

Ex 2:

$$\frac{4{,}424}{17.9-15.7} =$$

$$\frac{4{,}424}{17.9-15.7} = \frac{4{,}424}{2.2}$$

$$\begin{array}{r} 17.9 \\ -\ 15.7 \\ \hline 2.2 \end{array}$$

$$= 2010.9$$

$$= 2.0\times10^3$$

Problem:

The following are measured numbers. What is the product of 190.6 and 2.3?

+ & − has different rules than × & ÷

Solution

4 s.f. 2 s.f.

$190.6 \times 2.3 = 438.38$

keep 2 sf. drop these

$= 440$ or 4.4×10^2

Problem:

The following are measured numbers.
What is the sum of 125.17, 129 and 52.2?

+ & −
has different rules than
× & ÷

Solution

$$\begin{array}{r} 125.17 \\ 129 \\ +\ 52.2 \\ \hline 306.37 \end{array}$$

$$= 306$$

Problem:

The following are measured numbers. Calculate $\dfrac{15.035 - 14.966}{3.825}$

+ & −
has different rules than
× & ÷

Solution

$$\frac{15.035 - 14.966}{3.825} = 0.01803922$$

STEP 1:

$$\begin{array}{r} 15.035 \\ -\ 14.966 \\ \hline 0.069 \end{array}$$

STEP 2:

$$\frac{0.069}{3.825} = 0.01803922$$

$$= 0.018 \quad \text{or} \quad \boxed{1.8 \times 10^{-2}}$$

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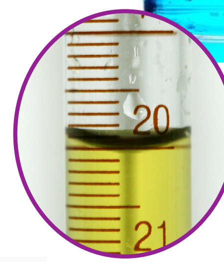

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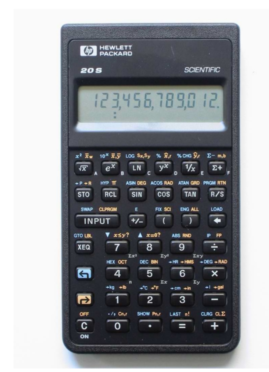

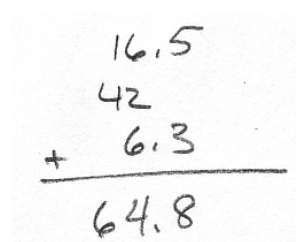

Questions?